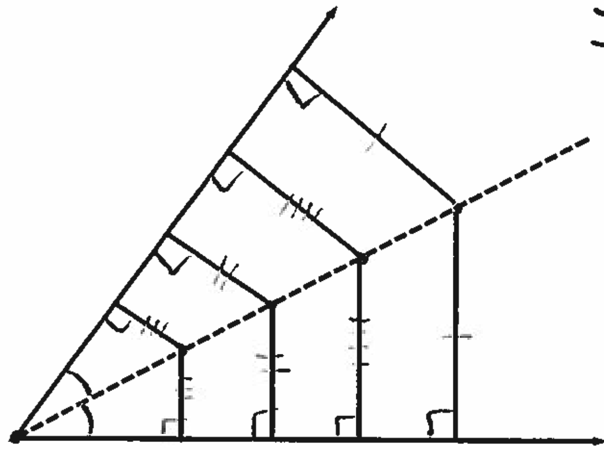
How many points are equidistant from the rays of an angle?



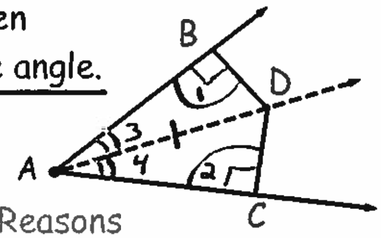
Infinitely many

This leads us to two more Theorems:

1. If a point lies on the bisector of an angle, then the point is equidistant from the sides of the angle.

2. If a point is equidistant from the sides of an angle, then the point lies on the bisector of the angle.

If a point lies on the bisector of an angle, then the point is equidistant from the sides of the angle.

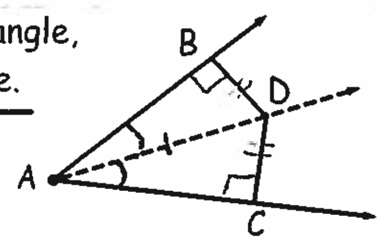


Given: $\overline{AD}$ bisects $\angle BAC$
$\overline{DB} \perp \overline{AB}, \overline{DC} \perp \overline{AC}$

Prove: $DB = DC$

Statements	Reasons
1 $\overline{AD}$ bisects $\angle BAC$ $\overline{DB} \perp \overline{AB}, \overline{DC} \perp \overline{AC}$	Given
2 $\angle 1$ and $\angle 2$ are Rt. $\angle$s	Def. of $\perp$
3 $\angle 1 \cong \angle 2$	Rt. $\angle$s Thm
4 $\overline{AD} \cong \overline{AD}$	Refl. Prop. of $\cong$
5 $\angle 3 \cong \angle 4$	Def. of $\angle$ bisector
6 $\triangle ACD \cong \triangle ABD$	AAS $\cong$ Thm
7 $\overline{DB} \cong \overline{DC}$	CPCTC
8 $DB = DC$	Def. of $\cong$ seg.

If a point is equidistant from the sides of an angle, then the point lies on the bisector of the angle.



Given: $DB = DC$ $\overline{DB} \perp \overline{AB}, \overline{DC} \perp \overline{AC}$

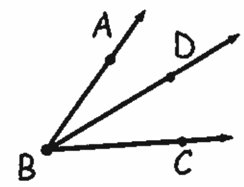
Prove: $\overrightarrow{AD}$ bisects $\angle BAC$

Statements	Reasons
1 $DB = DC$ $\overline{DB} \perp \overline{AB}, \overline{DC} \perp \overline{AC}$	Given
2 $\overline{AD} \cong \overline{AD}$	Refl. Prop. of $\cong$
3 $\overline{DB} \cong \overline{DC}$	Def. of $\cong$ seg.
4 $\triangle DAB \cong \triangle DAC$	HL $\cong$ Thm
5 $\angle BAD \cong \angle CAD$	CPCTC
6 $\overrightarrow{AD}$ bisects $\angle BAC$	Def. of $\angle$ bisector

1st Angle Bisector Theorem:

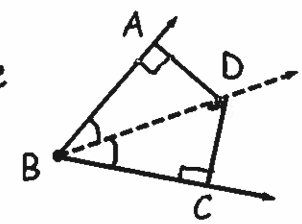
If $\overline{BD}$ is the angle bisector of $\angle ABC$ then

$$m\angle ABD = m\angle DBC = \frac{1}{2} m\angle ABC$$



2nd Angle Bisector Theorem:

A point lies on the bisector of an angle if and only if the point is equidistant from the sides of the angle.

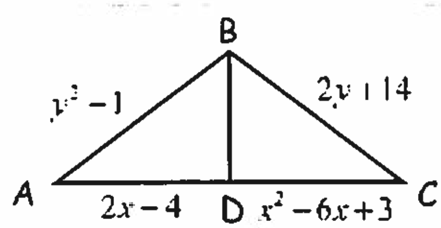


AD=DC

D is on bisector of $\angle B \longrightarrow AD = DC$

$AD = DC \rightarrow$ D is on the bisector of $\angle B$

$\overline{BD}$ is the $\perp$ bisector of $\overline{AC}$.
Find AB, BC, and AC.



① $AD = DC$ [⊥ bisector Thm]

$2x-4 = x^2-6x+3$

$0 = x^2-8x+7$

$0 = (x-1)(x-7)$

$x = 1, 7$

$x=1$ $\quad x=7$

$AC=-4$ X $\quad AC=20$

No neg lengths!

② $AB = BC$ [⊥ bisector Thm]

$y^2-1 = 2y+14$

$y^2-2y-15=0$

$(y-5)(y+3)=0$

$y=-3, 5$

$y=-3$ $\quad y=5$

$AB=8$ $\quad AB=24$

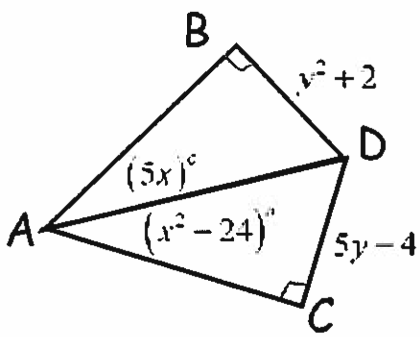
$BC=8$ $\quad BC=24$

20, 8, 8

X Will not make a Δ!

$\boxed{AB=24=BC \quad AC=20}$

$\overline{AD}$ is the angle bisector of $\angle BAC$. Find $m\angle BAC$ and DC.



① $m\angle BAD = m\angle CAD$ {∠ bisector Th #1}

$5x = x^2 - 24$

$0 = x^2 - 5x - 24$

$0 = (x-8)(x+3)$

$x = -3, 8$

$x = -3$

$m\angle BAC = -30°$

X No neg. measures!

$x = 8$

$\boxed{m\angle BAC = 80°}$

② $DC = BD$ {∠ bisector Th #2}

$5y - 4 = y^2 + 2$

$0 = y^2 - 5y + 6$

$0 = (y-3)(y-2)$

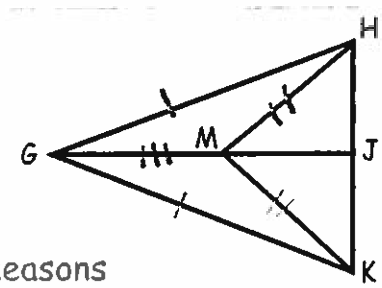
$y = 2, 3$

$y = 2$ $y = 3$

$\boxed{DC = 6 \text{ or } DC = 11}$

Given: $\overline{GJ}$ is the $\perp$ bisector of $\overline{HK}$

Prove: $\triangle GHM \cong \triangle GKM$



Statements	Reasons
1 $\overline{GJ}$ is the $\perp$ bisector of $\overline{HK}$	Given
2 $GH = GK$, $MH = MK$	$\perp$ bisector Thm
3 $\overline{GH} \cong \overline{GK}$, $\overline{MH} \cong \overline{MK}$	Def. of $\cong$ seg.
4 $\overline{GM} \cong \overline{GM}$	Refl. Prop. of $\cong$
5 $\triangle GHM \cong \triangle GKM$	SSS $\cong$ Post